Q.P. Code: 18HS0830 **R18**

Reg. No: | | | | | | | | | | |

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR

(AUTONOMOUS)

B.Tech. I Year I Semester Regular Examinations December 2018

MATHEMATICS-1

(Common to All)

Time: 3 hours Max. Marks: 60

PART-A

(Answer all the Questions 5 x 2 = 10 Marks)

1 **a** Define Symmetric & Skew-symmetric matrices. 2M

b Prove that $\Gamma(1)=1$. 2M

c If $\vec{f} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ find $div.\vec{f}$ at $(1,-1,1)$. 2M

d Define Power Series. 2M

e Find the Fourier coefficient a_0 when $f(x)=\frac{x}{2}, -\pi < x < \pi$. 2M

PART-B

(Answer all Five Units 5 x 10 = 50 Marks)

UNIT-I

2 Find the characteristic equation of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}.

Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. 10M

3 Find the Eigen values and corresponding Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. 10M

UNIT-II

4 **a** Find the volume of the sphere of radius 'a'. 5M

b Find the volume of the reel-shaped solid formed by the revolution about the *y*- axis, of the part of the parabola $y^2 = 4ax$ cut off by the latus- rectum. 5M

OR

5 **a** Verify Cauchy's mean value theorem for e^x and e^{-x} in (a,b). 5M

b Evaluate $\int_0^1 x^5 \left[log\left(\frac{1}{x}\right)\right]^3 dx$. 5M

UNIT-III

6 **a** If $u = tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 5M

b Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point $(2,-1,2)$. 5M

OR

7 **a** Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. 5M

b Show that $div\left(r^n \overline{r}\right) = (n+3)r^n$. 5M

UNIT-IV

8 Show that the series $1 + r + r^2 + r^3 + \text{-----}\infty$

i) Convergence if $|r| < 1$

ii) Divergence if $r \geq 1$

iii) Oscillates if $r \leq -1$. 10M

OR

9 Discuss the convergence of the series:

(i) $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + ---$

(ii) $1 + \frac{a+1}{b+1} + \frac{(a+1)(2a+1)}{(b+1)(2b+1)} + \frac{(a+1)(2a+1)(3a+1)}{(b+1)(2b+1)(3b+1)} + \ldots$ 10M

UNIT-V

10 Expand the function $f(x) = |x|$ in $-\pi < x < \pi$ as a Fourier series and

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} - - - - = \frac{\pi^2}{8}$. 10M

OR

11 Obtain the Fourier series expansion of $f(x) = 2x - x^2$ in $(0,3)$ and hence deduce that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots == \frac{\pi^2}{12}$. 10M

END